**Roanoke Valley Governor’s School for Science and Technology
Multivariable Calculus Competency List**

(Last updated: June, 2022)

Multivariable Calculus builds on the concepts learned in Accelerated Calculus (AP Calculus AB and AP Calculus BC) and is the equivalent of a third semester college calculus course. Students investigate the geometry of three-dimensional curves and surfaces and extend their knowledge of single-variable derivatives and integrals to three and more dimensions. The major topics include: vector valued functions, partial derivatives, multiple integrals, vector fields, line integrals, and Green’s Theorem.

(Dual enrollment course with Virginia Western Community College)

Students have the option of dual enrolling through Virginia Western Community College. They may earn a total of 4 credits by successfully completing both semesters of this course. The content of this course corresponds with Virginia Western’s MTH 265 Vector Calculus course.

This course is taught using best practices in gifted education. Each competency is aligned with Hockett’s five principles of gifted education:

**Gifted Education Principles:**( Hockett, J.A. (2009) “Curriculum for Highly Able Learners That Conforms to General Education and Gifted Education Quality Indicators.” *Journal of Education for the Gifted***. Vol. 32, No. 3, p. 394-440)**

1. High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.
2. High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.
3. High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional.
4. High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true to life, and outcomes that are transformational.
5. High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety.

*External standards from Virginia Community College Standards were referenced when reviewing these competencies. To the right of each Enabling Objective is notation indicating alignment with external standards and a relative priority/proficiency rating from A (highest) to D (lowest).*

COMPETENCY I

**Perform operations on vectors and use vectors to interpret geometry in three-dimensional space.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Interpret equations and inequalities geometrically in the xyz planes (space).
 | 1.1 | A |
| 1. Calculate the distance between points in the space.
 | 1.1 | A |
| 1. Calculate the magnitude of a three-dimensional vector.
 | 1.2 | A |
| 1. Perform vector addition and scalar multiplication on vectors.
 | 1.3 | B |
| 1. Find the angle between two nonzero vectors. Determine whether vectors are parallel, perpendicular, or neither.
 | 1.4 | A |
| 1. Determine scalar and vector projections
 | 1.5 | C |
| 1. Find the dot product of two vectors.
 | 1.4 | A |
| 1. Find the cross product of two vectors.
 | 1.4 | A |
| 1. Write a set of parametric equations for a line in space.
 | 1.6 | A |
| 1. Write a linear equation to represent a plane in space.
 | 1.6 | A |
| 1. Find the distance between points and planes, points and lines, two planes, and two lines in space.
 | Exceeds standard | C |
| 1. Identify various quadric surfaces by equation and by their graphs using traces and cross sections.
 | 1.7 | B |

COMPETENCY II

**Analyze and apply vector-valued functions to describe motion in space, based upon tangent and normal vectors and the principles of curvature.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| * 1. Define and sketch (2d) a vector-valued functions.
 | 2.1-2.2 | A |
| * 1. Extend the concepts of limits and continuity to vector-valued functions.
 | 2.3 | A |
| * 1. Differentiate a vector-valued function.
 | 2.3 | A |
| * 1. Integrate a vector-valued function.
 | 2.3 | A |
| * 1. Apply vector-valued functions to interpret projectile motion.
 | 2.6 | B |
| * 1. Determine the length of a curve in three-dimensional space.
 | 2.4 | A |
| * 1. Find a curve’s unit tangent vector.
 | 2.4 | A |
| * 1. Find the curvature of a plane and space curves.
 | 2.4 | A |
| * 1. Find the principal unit normal vector for plane and space curves.
 | 2.4 | A |
| * 1. Find the binormal vector for space curves.
 | 2.4 | A |
| * 1. Find the tangential and normal components of acceleration.
 | 2.6 | C |

COMPETENCY III

**Find limits, determine continuity, and find partial derivatives for functions of several variables.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| * 1. Interpret the notation for a function of several variables.
 | 3.1 | A |
| * 1. Determine the domain and range of a function of two and three variables.
 | 3.1 | A |
| * 1. Define interior and boundary points.
 | 3.1 | C |
| * 1. Determine if a region is open or closed using interior and boundary points.
 | 3.1 | C |
| * 1. Generate level curves and contour curves for a function of several variables. Use them to interpret the relationship between level curves and extrema and level curves and gradient vectors.
 | Exceeds standard | B |
| * 1. Evaluate the limit for a function of two variables.
 | 3.2 | B |
| * 1. Extend the definition of continuity to functions of two and three variables.
 | 3.3 | B |
| * 1. Apply properties of limits to functions of two variables.
 | 3.3 | B |
| * 1. Find and use partial derivatives of a function of two variables.
 | 3.4 | A |
| * 1. Find and use partial derivatives of a function of three or more variables.
 | 3.4 | A |
| * 1. Find higher-order partial derivatives of a function of two or more variables.
 | 3.4 | A |
| * 1. Use chain rule for functions of several variables.
 | 3.6 | A |
| * 1. Find partial derivatives implicitly.
 | 3.4 | B |

COMPETENCY IV

**Apply knowledge of partial derivatives to find directional derivatives, tangent planes, and extrema on functions of several variables.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| * 1. Find and use directional derivatives of a function of two variables.
 | 3.7 | A |
| * 1. Find the gradient of a function of two variables.
 | 3.8 | A |
| * 1. Use the gradient of a function of two variables in applications.
 | 3.8 | C |
| * 1. Find directional derivatives and gradients of functions of three variables.
 | 3.8 | A |
| * 1. Find equations of tangent planes and normal lines to surfaces.
 | 3.5 | B |
| * 1. Find global and local extrema of a function of two variables.
 | 3.9 | B |
| * 1. Use the Second Partials Test to find local extrema of a function of two variables.
 | 3.9 | A |
| * 1. Use the method of Lagrange multipliers to find extrema over a particular constraint.
 | 3.9 | C |

COMPETENCY V

**Evaluate an integral of a function of two variables in a plane and three variables over a region in space. Apply these concepts to determine volume, areas in a plane, moments, and centers of mass.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Define a double integral
 | 4.1 | A |
| 1. Evaluate an iterated integral using Fubini’s Theorem.
 | 4.2 | A |
| 1. Use an iterated integral to find the area of a plane region.
 | 4.3 | B |
| 1. Use a double integral to represent the volume of a region.
 | 4.3 | B |
| 1. Use properties of double integrals.
 | 4.2 | A |
| 1. Find the average value of a function over a region.
 | Exceeds standard | C |
| 1. Evaluate a triple integral as an iterated integral using Fubini’s Theorem.
 | 4.5 | A |
| 1. Write and evaluate double integrals in polar coordinates.
 | 4.4 | B |
| 1. Use a triple integral to find the volume of a solid region.
 | 4.5 | A |
| 1. Explore applications of double/triple integrals such as the center of mass and moments of inertia of a solid region.
 | 4.3 | C |
| 1. Understand the concept of a Jacobian.
 | 4.6 | C |
| 1. Use a Jacobian to change variables in a double integral.
 | 4.6 | C |

COMPETENCY VI

**Evaluate line and surface integrals and investigate their applications to science and engineering.**

*Enabling Objectives:*

|  |  |  |
| --- | --- | --- |
| 1. Compute line integrals for scalar functions.
 | 5.2 | A |
| 1. Compute line integrals for vector-valued functions.
 | 5.2 | A |
| 1. Understand the concept of a vector field.
 | 5.1 | A |
| 1. Determine whether a vector field is conservative.
 | 5.1 | A |
| 1. Find the curl of a vector field.
 | 5.5 | B |
| 1. Find the divergence of a vector field.
 | 5.5 | B |
| 1. Find potential function for a vector field.
 | 5.3 | B |
| 1. Apply the connection between concepts of conservative force field, independence of path, and the existence of potential functions.
 | 5.3 | B |
| 1. Understand and use the Fundamental Theorem of Line Integrals.
 | 5.3 | A |
| 1. Use Green’s Theorem to evaluate a line integral.
 | 5.4 | B |
| 1. Evaluate a surface integral.
 | 5.6 | C |
| 1. Compute and understand the concept of flux of a vector field.
 | 5.6 | D |
| 1. State and use Stokes Theorem
 | 5.7 | D |
| 1. State and use the Divergence Theorem
 | 5.7 | D |